

Introduction to multivariate analysis

— applications in genomics —

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MSc “*Modern epidemiology*”

22-03-2013

Outline

Multivariate analysis in a nutshell

Applications to genomic data

Genetic diversity of pathogen populations

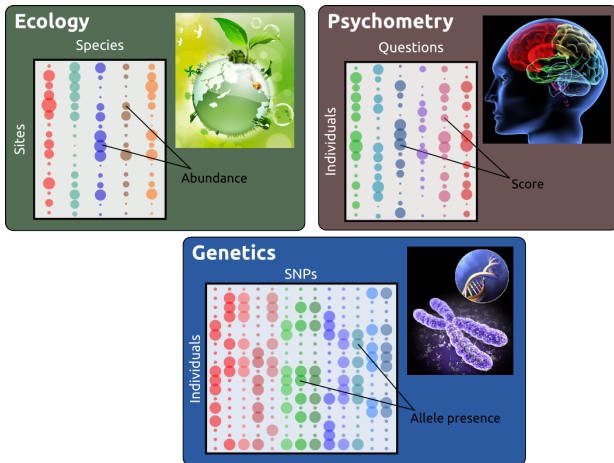
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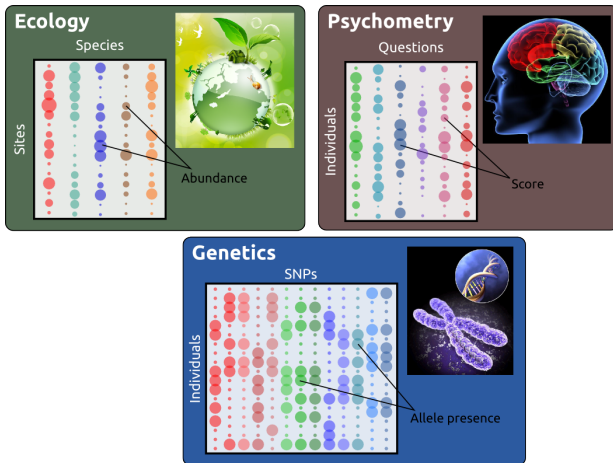
Genetic diversity of pathogen populations

Multivariate data: some examples



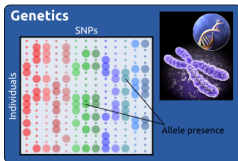
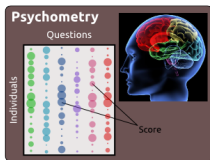
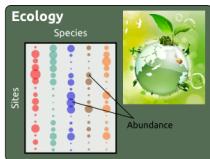
Association between individuals? Correlations between variables?

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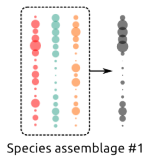
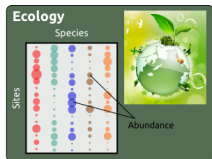


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Multivariate analysis to summarize diversity

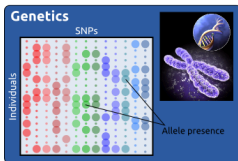
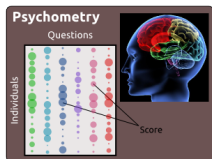


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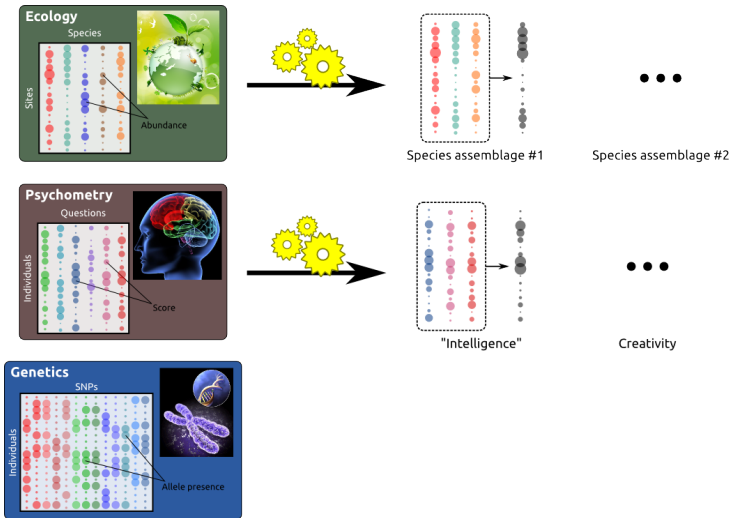


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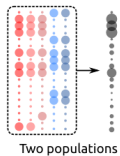
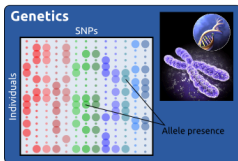
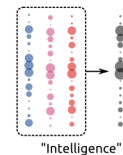
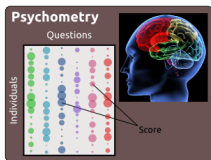
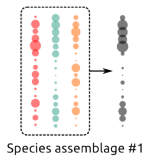
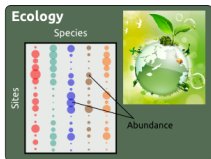
Species assemblage #2



Multivariate analysis to summarize diversity



Multivariate analysis to summarize diversity



Multivariate analysis: an overview

Multivariate analysis, a.k.a:

- “*dimension reduction techniques*”
- “*ordinations in reduced space*”
- “*factorial methods*”

Purposes:

- summarize diversity amongst observations
- summarize correlations between variables

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Most common methods

Differences lie in input data:

- quantitative/binary variables: *Principal Component Analysis* (PCA)
- 2 categorical variables: *Correspondance Analysis* (CA)
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- Euclidean distance matrix: *Principal Coordinates Analysis* (PCoA) / *Metric Multidimensional Scaling* (MDS)

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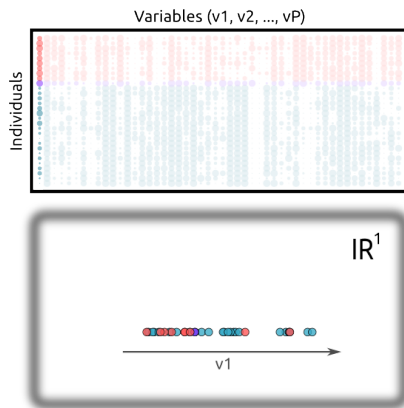
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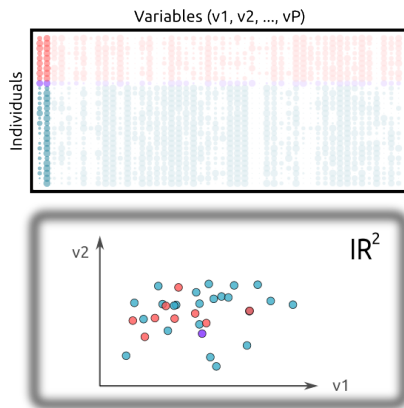
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1 dimension, 2 dimensions, P dimensions



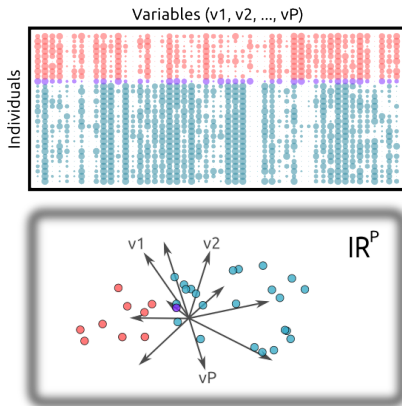
Need to find most informative directions in a P -dimensional space.

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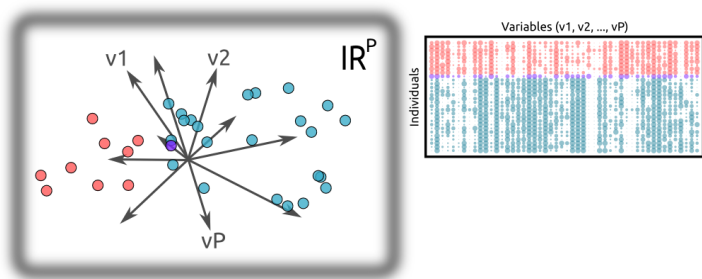
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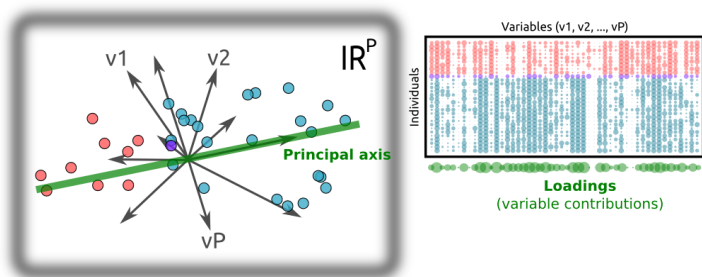
Reducing P dimensions into 1



- $\mathbf{X} \in \mathbb{R}^{N \times P}$; $\mathbf{X} = [\mathbf{x}_1 | \dots | \mathbf{x}_P]$: data matrix
- $\mathbf{u} \in \mathbb{R}^P$; $\mathbf{u} = [u_1, \dots, u_P]$: **principal axis**
($\|\mathbf{u}\|^2 = \sum_{j=1}^P u_j^2 = 1$)
- $\mathbf{v} \in \mathbb{R}^N$; $\mathbf{v} = \mathbf{X}\mathbf{u} = \sum_{j=1}^P u_j \mathbf{x}_j$: **principal component**

→ find \mathbf{u} so that $\frac{1}{N} \|\mathbf{v}\|^2 = \text{var}(\mathbf{v})$ is maximum.

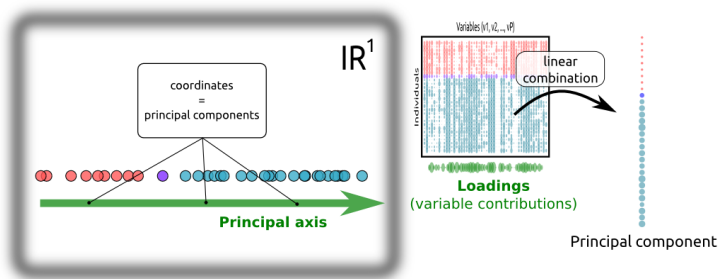
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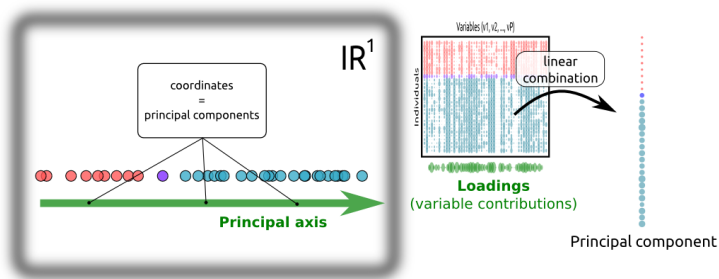
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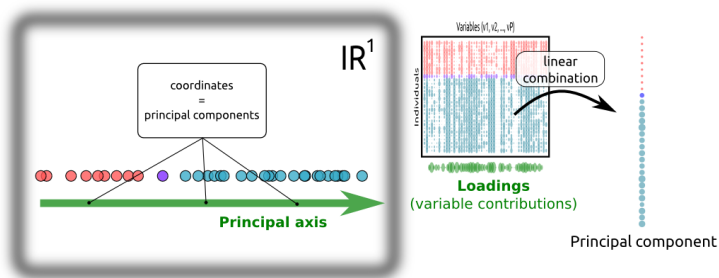
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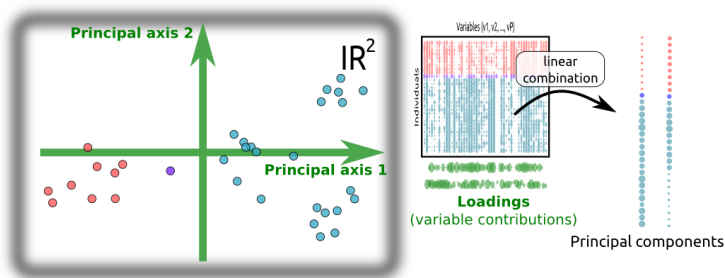


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- \mathbf{u}_2 and \mathbf{v}_2 : **2nd principal axis and component**

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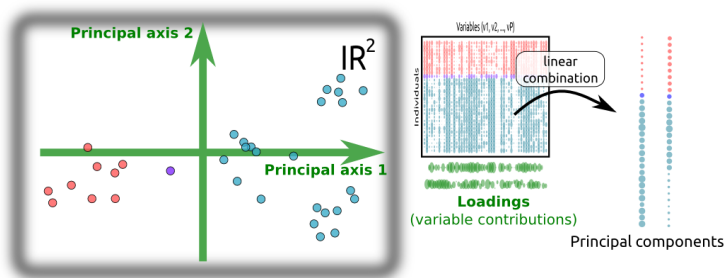


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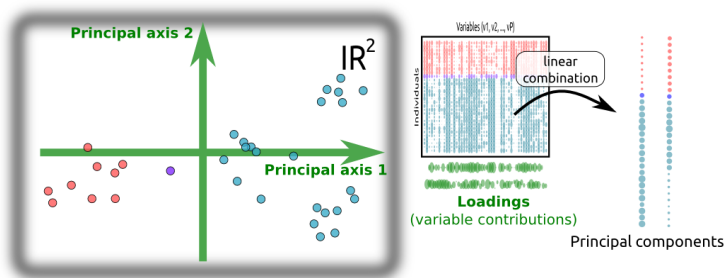


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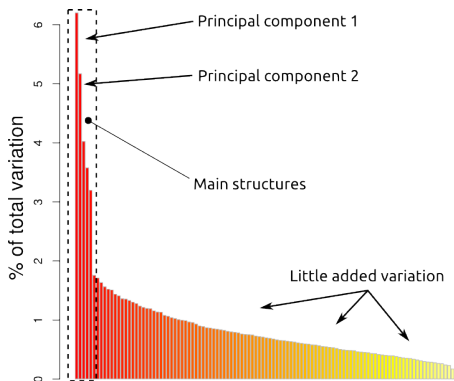
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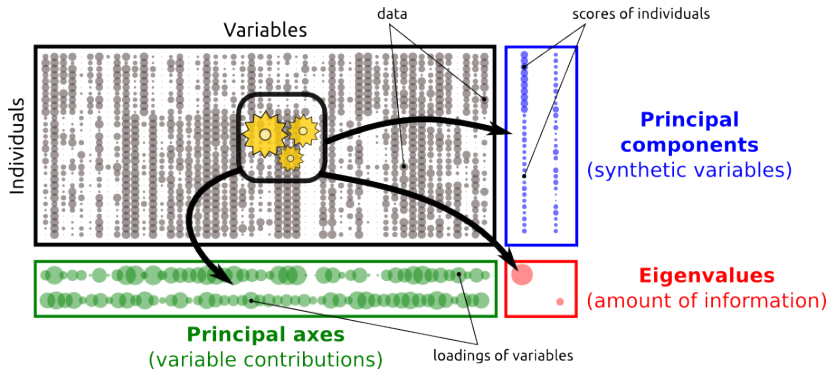
How many principal components to retain?

Choice based on “**screplot**”: barplot of eigenvalues



Retain only “significant” structures... but not trivial ones.

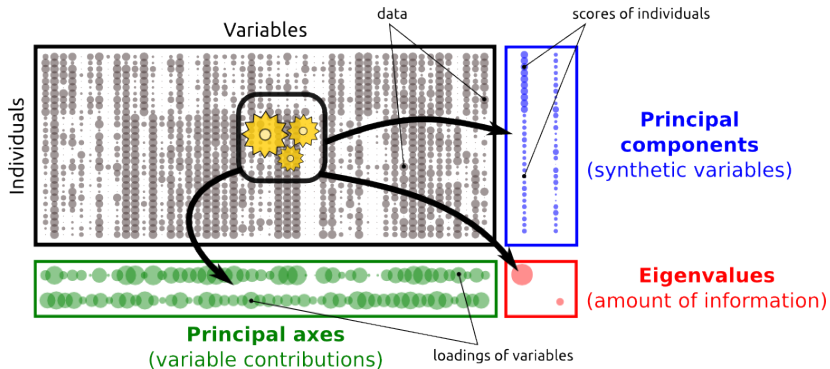
Outputs of multivariate analyses: an overview



Main outputs:

- **principal components:** diversity amongst individuals
- **principal axes:** nature of the structures
- **eigenvalues:** magnitude of structures

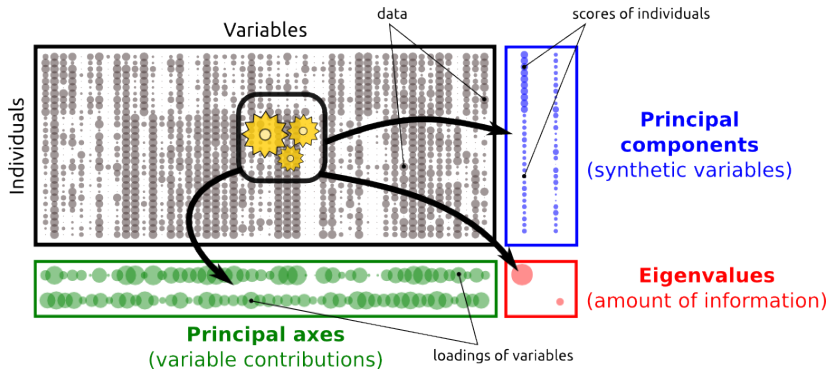
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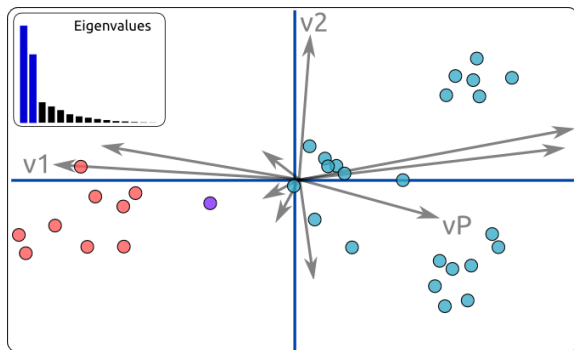
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Usual summary of an analysis: the biplot



Biplot: principal values (points) + loadings (arrows)

- groups of individuals
- discriminating variables (longest arrows)
- magnitude of the structures

Multivariate analysis in a nutshell

- **variety of methods** for different types of variables
- **principal components** (PCs) summarize diversity
- **variable loadings** identify discriminating variables
- other uses of PCs: **maps** (spatial structures), **models** (response variables or predictors), ...

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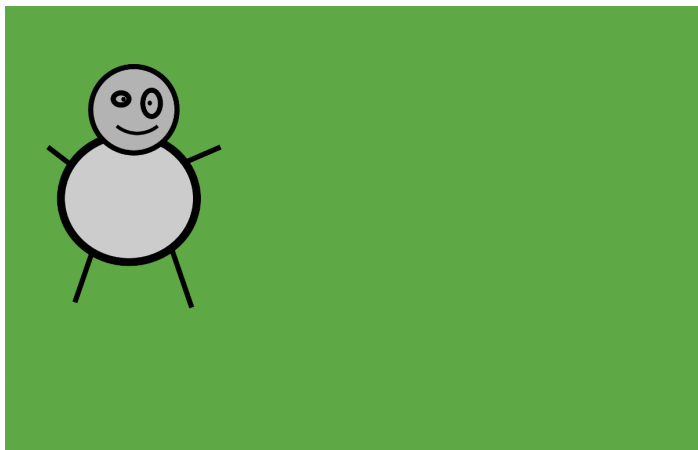
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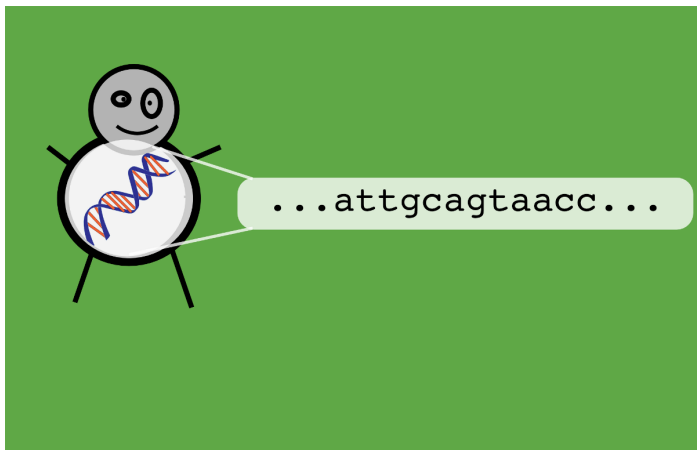
Applications to genomic data

Genetic diversity of pathogen populations

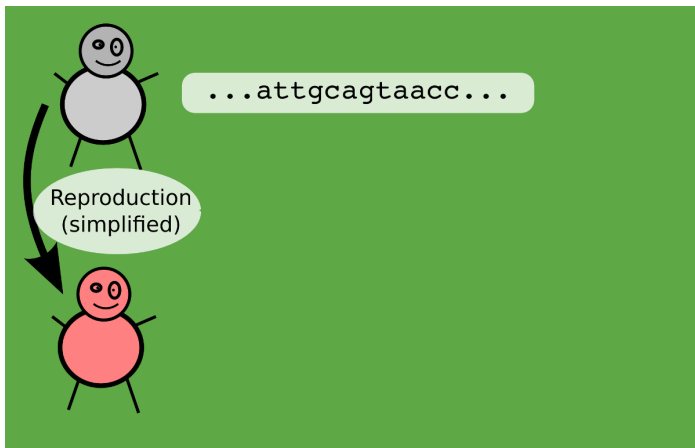
From DNA sequences to patterns of biological diversity



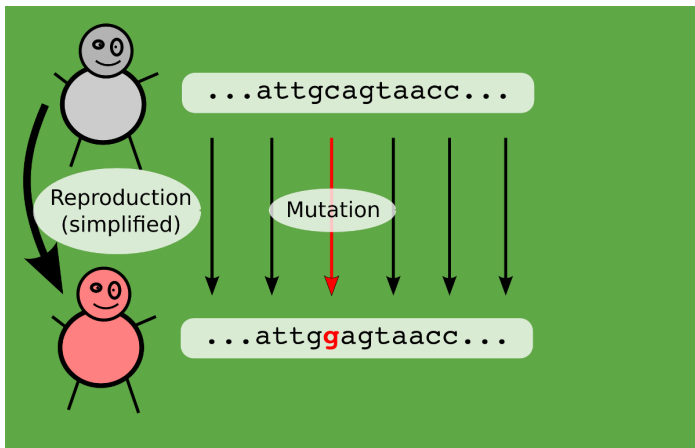
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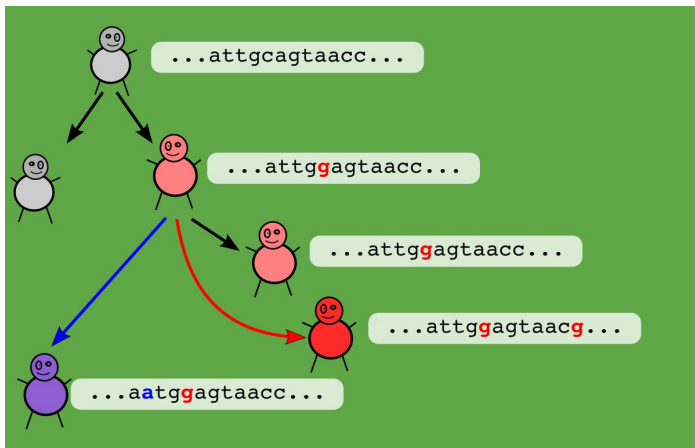
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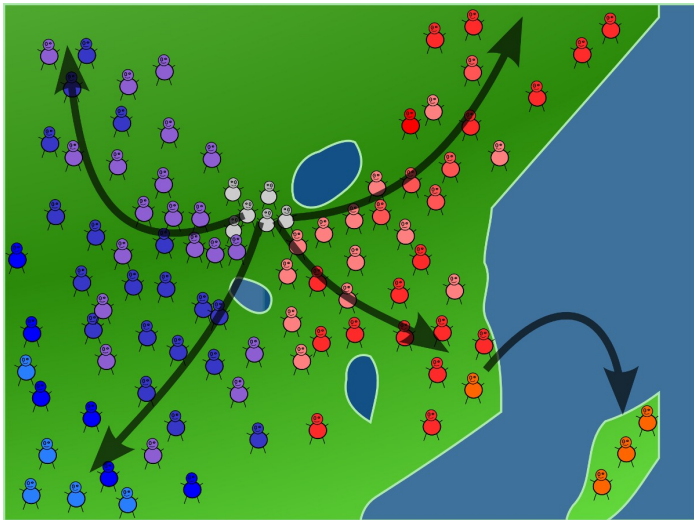
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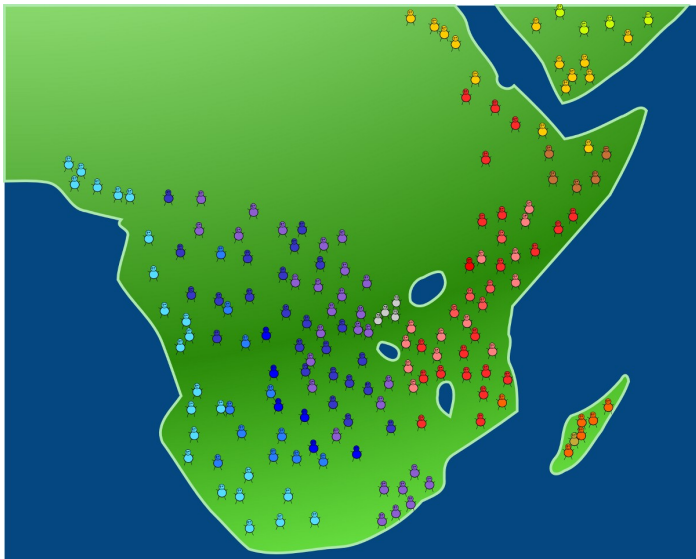
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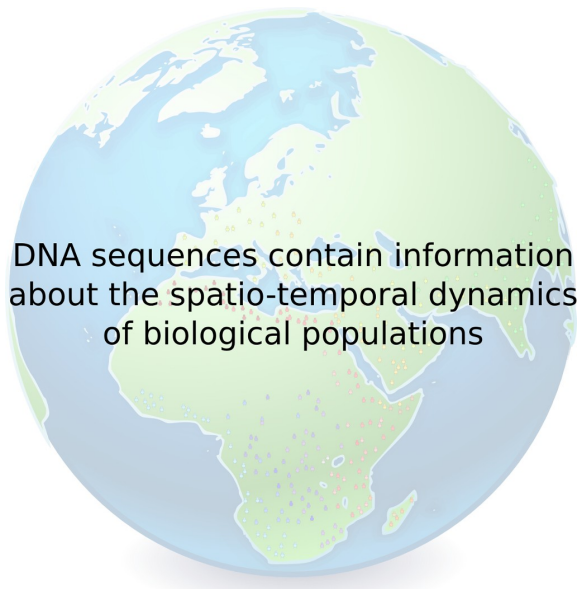
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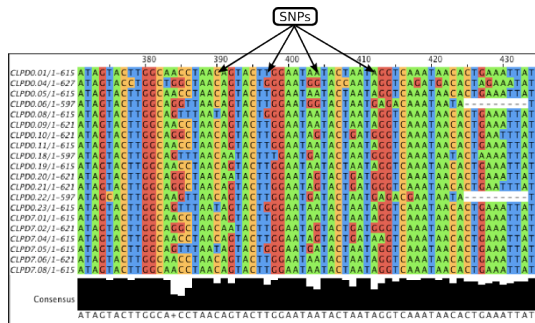


From DNA sequences to patterns of biological diversity

A 3D rendering of the Earth, showing continents in light green and oceans in light blue. The globe is centered on the African continent. Numerous small, multi-colored dots (red, blue, yellow, green) are scattered across the landmasses, representing sampling locations for DNA sequences. The text is overlaid on the center of the globe.

DNA sequences contain information
about the spatio-temporal dynamics
of biological populations

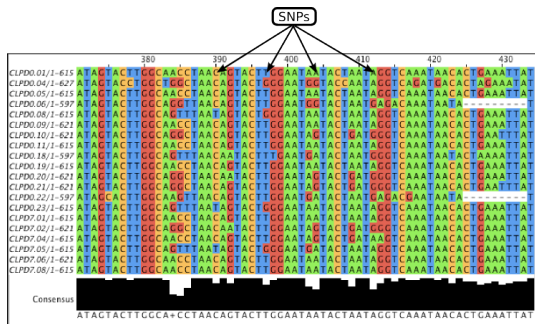
DNA sequences: a rich source of information



- hundreds/thousands individuals
- up to millions of single nucleotide polymorphism (SNPs)

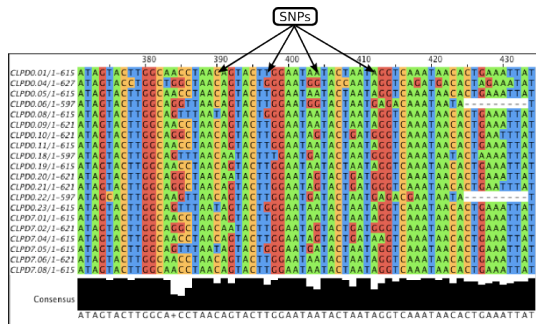
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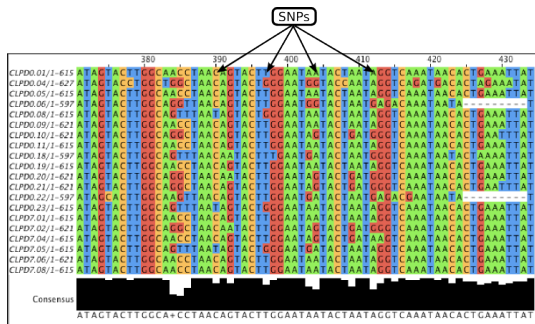
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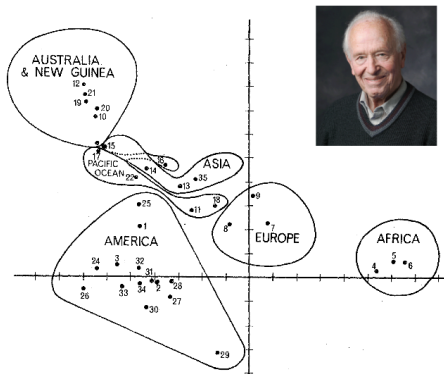
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First application of multivariate analysis in genetics

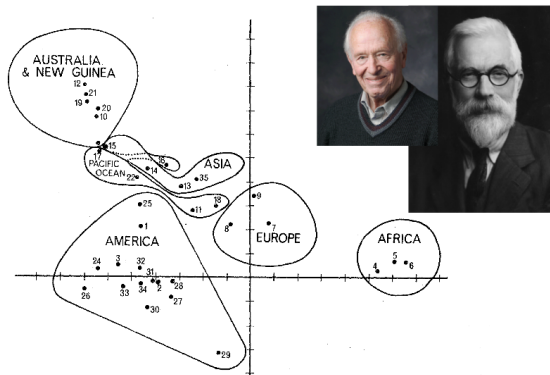
PCA of genetic data, native human populations (Cavalli-Sforza 1966, *Proc B*)



First 2 principal components separate populations into continents.

First application of multivariate analysis in genetics

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First 2 principal components separate populations into continents.

Applications: some examples

PCA of genetic data + colored maps of principal components

(Cavalli-Sforza et al. 1993, *Science*)



Signatures of Human expansion out-of-Africa.

Since then...

Multivariate methods used in genetics

- Principal Component Analysis (PCA)
- Principal Coordinates Analysis (PCoA) / Metric Multidimensional Scaling (MDS)
- Correspondance Analysis (CA)
- Discriminant Analysis (DA)
- Canonical Correlation Analysis (CCA)
- ...

Since then...

Applications

- reveal spatial structures (historical spread)
- explore genetic diversity
- identify cryptic species
- discover genotype-phenotype association
- ...
- review in Jombart et al. 2009, *Heredity* **102**: 330-341

Applications in genetics of pathogen populations.

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Why investigate the diversity of pathogen populations?

Genetic data: increasingly important in infectious disease epidemiology

Purposes

- classify pathogens, describe their relationships
- assess the spatio-temporal dynamics of infectious diseases
- reconstruct epidemiological processes (transmission)



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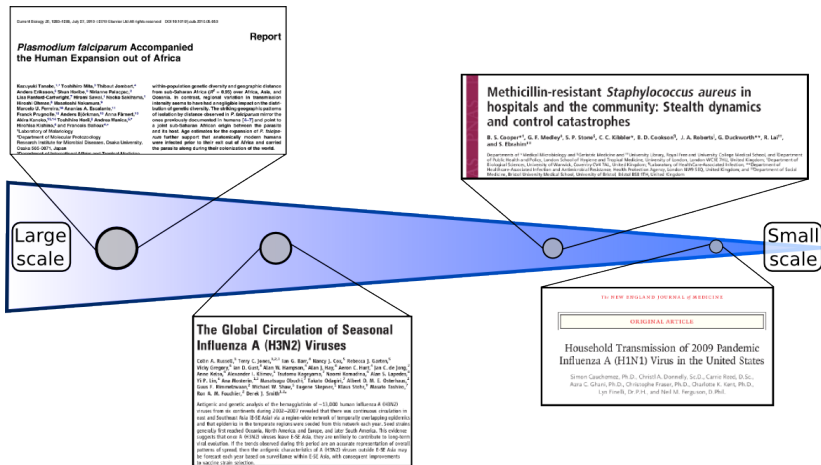
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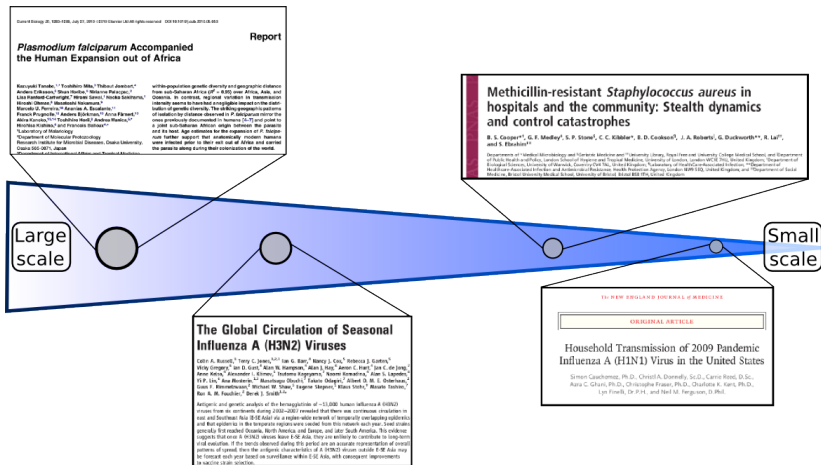


Different questions at different scales



Where and how can multivariate analysis of pathogen genetic data be useful?

Different questions at different scales



Where and how can multivariate analysis of pathogen genetic data be useful?

Describing pathogen populations

Population genetics: identify populations of organisms and describe their relationships

What is a population?

- *Usual definition:* set of organisms mating at random
- *Problem:* no “mating” in most pathogens (e.g. viruses, bacteria)
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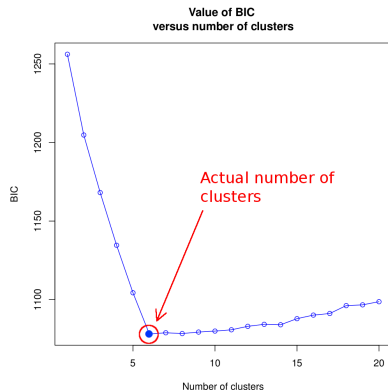
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Genetic clustering using K-means & BIC

(Jombart *et al.* 2010, *BMC Genetics*)

Variance partitioning model (ANOVA):

$$\text{tot. variance} = (\text{bet. groups}) + (\text{wit. groups})$$



Performances:

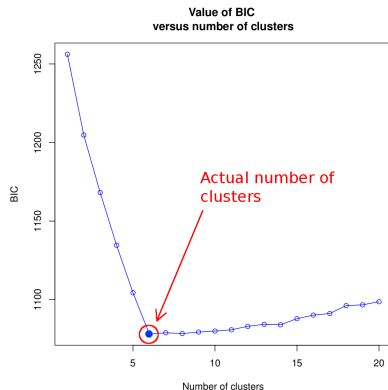
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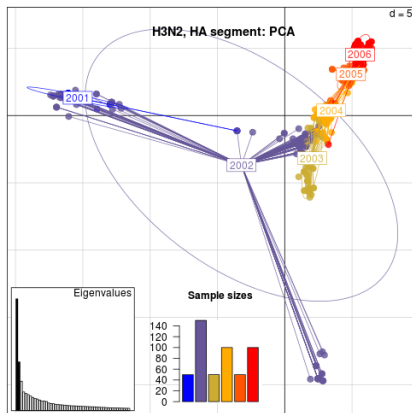


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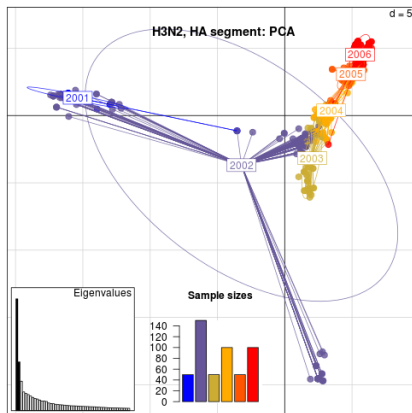
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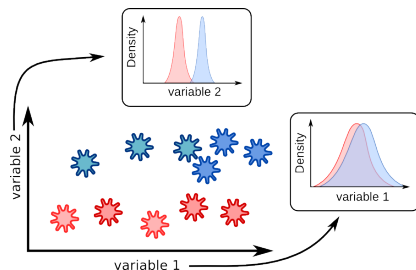
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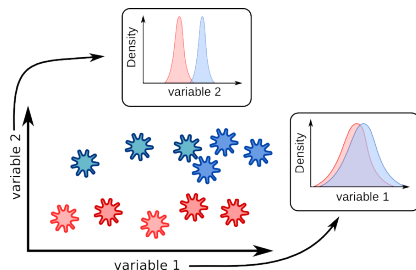
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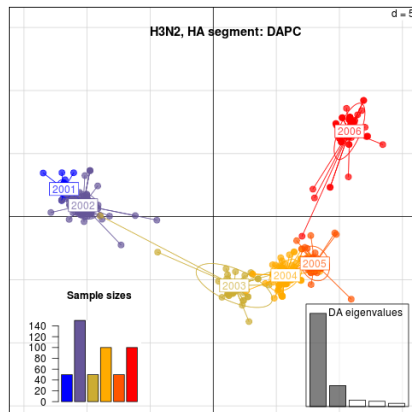


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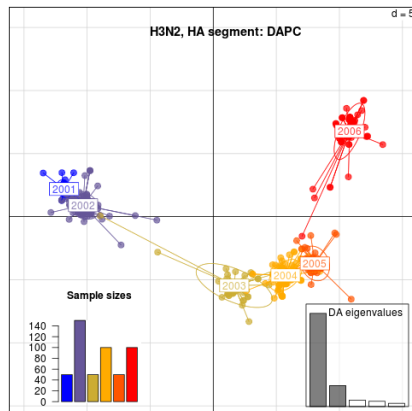
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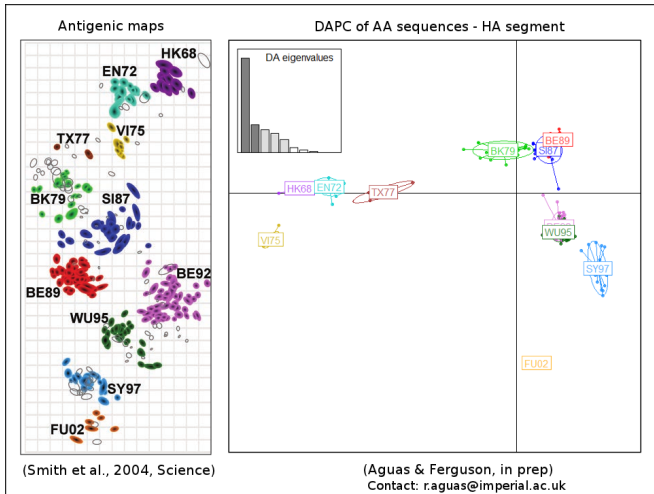
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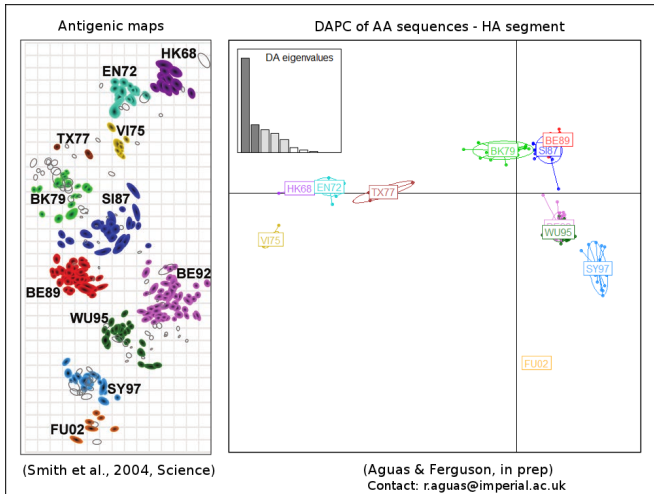
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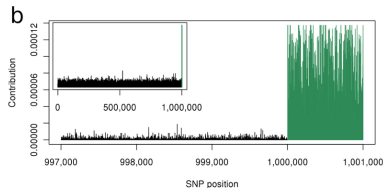
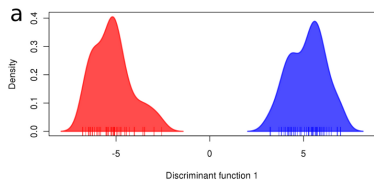
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DAPC finds combinations of alleles most differing between groups.

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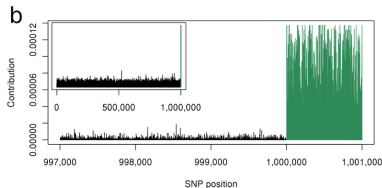
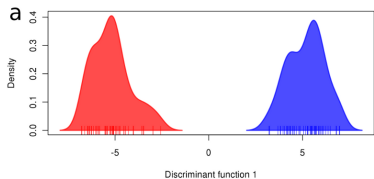
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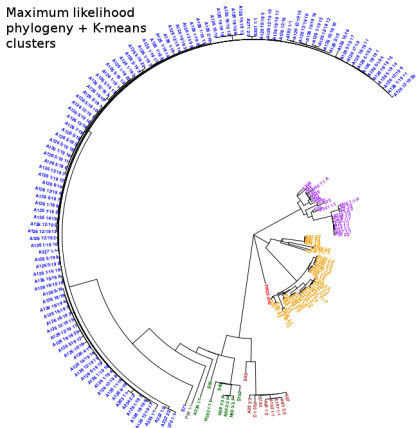


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phylogeny + K-means
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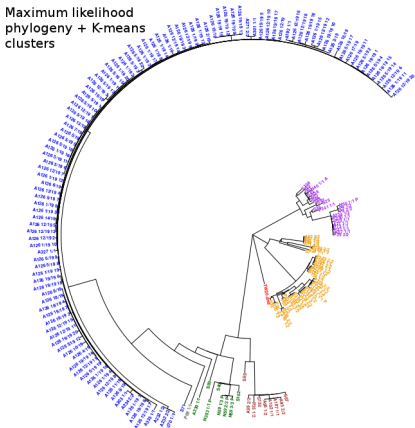
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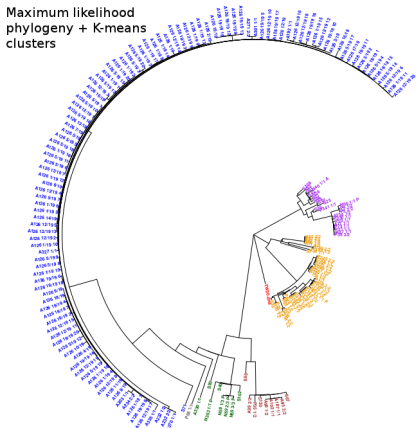
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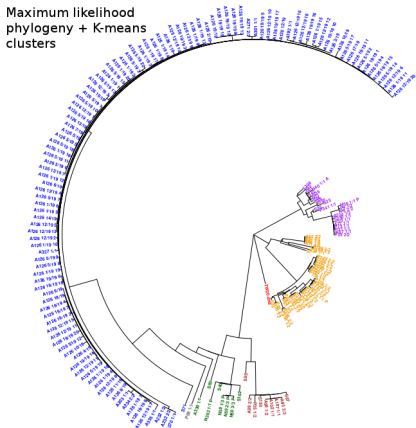
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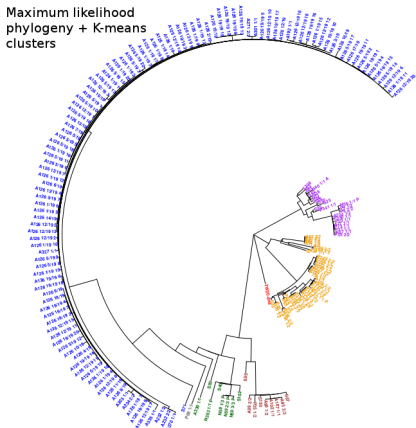
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Multivariate analysis usually not informative on small-scale processes.

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